The Handlos-Baron Drop Extraction Model

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The Handlos-Baron drop extraction model (*1*) and the Stokes-Einstein equation for molecular diffusion bear a curious inverse relationship to each other: the latter utilizes the continuum model of the hydrodynamics of a sphere moving in a fluid to predict the molecular parameter of the diffusion coefficient, while the former employs the particlelike properties of Brownian motion as a hydrodynamic basis for predicting the macroscopic characteristics of internal solute transfer in falling drops.

Despite the ad hoc nature of the underlying physical concept, the Handlos-Baron model is the only extant theory for dispersed phase mass transfer in turbulent drops. The accuracy of its mathematical development is therefore of some interest. Consideration is restricted to the constant velocity regime.

The consequences of the Handlos-Baron model are contained in the equation

$$\frac{\partial C}{\partial \tau} = \frac{N_{Pe}}{128} \frac{1}{\eta} \frac{\partial}{\partial \eta} \left[g(\eta)\, \eta \frac{\partial C}{\partial \eta} \right] \quad (1)$$

where N_{Pe}, the Peclet number, is shown to be of importance in nonoscillating drops (*2*), and $g(\eta)$ is the function

$$g(\eta) = 3 - 8\eta + 6\eta^2 \quad (2)$$

The initial and boundary conditions on Equation (1) are

$$\begin{aligned} C(0, \eta) &= 1 \\ C(\tau, 1) &= 0 \\ (\partial C/\partial \eta)_o &= 0 \end{aligned} \quad (3)$$

Handlos and Baron solved Equation (1) by an eigenfunction expansion, of which they retained only the first term:

$$E_m = 1 - \overline{C} = 1 - \exp\left\{ - \frac{\lambda_1 N_{Pe}\, \tau}{128} \right\} \quad (4)$$

where λ_1 was found to be equal to 2.88 [later recalculated as 2.866 by Wellek and Skelland (*4*)].

In terms of a mass transfer coefficient defined by

$$E_m = 1 - \exp\left\{ - \frac{3kt}{a} \right\} \quad (5)$$

the Handlos-Baron solution yields

$$k_{HB} = \frac{0.00375\, u}{1 + \mu_D/\mu_C} \quad (6)$$

In order to verify Equation (4), Equation (1) and its associated conditions were solved numerically. The differential equation was written in finite-difference form and numerical values of the total amount of solute remaining were obtained as a function of time. The results shown in Table 1 are based upon an interval of the dimensionless time in the first column of Table 1 of 0.0005 and an η interval of 0.0125. Except for dimensionless times less than 0.01, these results were within 1% of the results with time and distance intervals of 0.001 and 0.025, respectively. It can be seen that the fraction extracted predicted by the Handlos-Baron solution underestimates the consequences of the Handlos-Baron model by substantial amounts at small values of the dimensionless time.

Table 1. Solutions of the Handlos-Baron Model

$\frac{1}{128} N_{Pe}\tau$	Fraction extracted: Numerical	Fraction extracted: Equation (4)
0.01	0.196	0.028
0.05	0.384	0.134
0.10	0.496	0.250
0.15	0.572	0.351
0.20	0.632	0.438
0.30	0.724	0.579
0.40	0.791	0.684
0.50	0.842	0.763
0.70	0.910	0.867
1.00	0.961	0.944

For values greater than 0.1 in the first column of Table 1, the numerical solution can be very well approximated by

$$E_m = 1 - 0.64 \exp\left\{ - \frac{2.80\, N_{Pe}\, \tau}{128} \right\} \quad (7)$$

Or, in terms of the mass transfer coefficient defined by Equation (5)

$$k = 0.972\, k_{HB} + 0.15 \frac{a}{t} \quad (8)$$

In dimensionless terms, Equation (8) is

$$N_{Sh} = 0.972\, N_{ShHB} + \frac{0.3}{\tau} \quad (9)$$

The major difference between the numerical and eigenvalue results is in the coefficient of the exponential term in Equations (4) and (7). Handlos and Baron retained only the first term of the series and to fit the initial condition set the coefficient equal to unity. However the coefficient of the first term is 0.64, which adds to the equation for the mass transfer coefficient a time-dependent term that becomes important for large drops or small contact times. The dependence of the dispersed phase mass transfer coefficient on the contact time has been experimentally observed by Skelland and Wellek (*3*).

ACKNOWLEDGMENT

This work was carried out as a part of the EURATOM research program, with cooperation from the Computer Center at Ispra.

NOTATION

- a = drop radius
- C = solute concentration
- $\overline{C}$ = average concentration in drop
- D = diffusion coefficient in drop

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E_m = fraction extracted
k = dispersed phase mass transfer coefficient
N_{Pe} = Peclet number, $\frac{1}{2}\frac{\mu_C}{\mu_C+\mu_D}\frac{ua}{D}$
N_{Sh} = Sherwood number, $2ka/D$
r = radial position in Handlos-Baron model
t = contact time
u = drop velocity
τ = dimensionless time, Dt/a^2
η = dimensionless position, $r/(a/2)$
μ = viscosity of drop liquid

Subscripts

c = continuous phase
D = dispersed phase
HB = based on Handlos-Baron solution

LITERATURE CITED

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Application of Integral Momentum Methods to Viscoelastic Fluids: Flow About Submerged Objects

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Viscoelastic flow problems involving the drag resistance of dilute polymer solutions have in recent years become of great interest to the petroleum industry and in naval applications (*4, 8, 12, 16, 20*). The rapid laminar flow of slightly viscoelastic fluids about submerged objects was first studied by Rajeswari and Rathna (*19*), who formulated the two-dimensional boundary-layer equations and looked in particular at the problem of flow near a stagnation point. Rajeswari and Rathna represented the stress field with the Coleman-Noll second-order fluid (*9, 15, 26*). Essentially the same problem was reanalyzed by Beard and Walters (*2*) in a somewhat different fashion. A different attack on viscoelastic boundary-layer theory has been made by White and Metzner (*27, 28*), who noting that the second-order fluid concept is inadequate at the high deformation rates involved in the boundary-layer problem, developed a more complex constitutive equation which introduces non-Newtonian viscosity as well as second-order viscoelastic effects. They go on to indicate the range of validity of viscoelastic boundary-layer theory and relate it specifically to the theory of purely viscous non-Newtonian boundary layers developed by Schowalter (*22*) and by Acrivos, Shah, and Petersen (*1, 23*) (see also reference *11*). Essentially it was found that these purely viscous solutions are valid up to rather high values of the Weissenberg number* based upon the boundary-layer thickness, but to only small values of Weissenberg numbers based upon the distance along the surface from the forward stagnation point. High boundary-layer thickness Weissenberg numbers imply that highly elastic behavior is required to produce any measurable effects in this flow field. In the range of such large Weissenberg numbers, White and Metzner found several external velocity fields which would allow transformation of the boundary-layer equation to an ordinary differential equation, but the problem of greatest interest, the constant mainstream velocity, cannot be so transformed. Recently, Denn (*10*) generalized the work of White and Metzner by using a more complex constitutive law and obtained additional transformation solutions. However Denn was not able to obtain a transformation for the constant mainstream velocity case.

There exists an alternative approach to this problem in which the equations of motion are integrated in a special manner. This integral momentum method for Newtonian fluids is reviewed by Schlichting (*21*); it was introduced into non-Newtonian fluid dynamics by Bogue (*5*) and by Acrivos, Shah, and Petersen (*1*). The flow of purely viscous non-Newtonian fluids about submerged objects has been studied by this procedure by Acrivos, Shah, and Petersen (*1, 23*). Integral momentum procedures were introduced into viscoelastic fluid dynamics by Metzner and White (*17*) in order to study the entrance region. Their use of a modified von Karman method suggests a unique approach to many viscoelastic flow problems, for it enables one to obtain approximate solutions without presuming a particular constitutive equation for the stress. Rather, one may directly use viscometric laminar shear flow relationships between the shear and normal stress components and the shear rate. The saving in effort by this method is considerable; one need only read the papers of White and Metzner (*27*) and Denn (*10*) to see this. In this communication, we will apply this perhaps unique procedure to the flow past a semi-infinite flat plate.

EQUATIONS OF MOTION

Consider the rapid flow of an infinite viscoelastic fluid with a mainstream velocity $U(x)$ past a submerged flat plate, which stretches from the origin along the x axis. The fluid adheres to the flat plate and its velocity varies rapidly through a distance $\delta(x)$ to the mainstream value. The equations of motion may be simplified by the Prandtl kinematic approximations:

$$u \sim U, \quad v \sim U\,\delta/x, \quad \frac{\partial u}{\partial y} \sim \frac{U}{\delta}, \quad \text{etc.}$$

This allows us to rewrite the equations of motion for values of y less than $\delta(x)$ as [for Newtonian fluids see Schlichting (*21*) and for viscoelastic fluids see White and Metzner (*27*) and Rajeswari and Rathna (19)]:

$$\rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial p}{\partial x} + \frac{\partial t_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \quad (1a)$$

* The Weissenberg number, named for Karl Weissenberg, who was one of the pioneers in nonlinear viscoelastic hydrodynamics, represents a ratio of viscoelastic to viscous forces in a flow field. Applications of this dimensionless group to different theoretical and industrial problems are discussed in the literature (*26* to *28, 17, 4, 18, 10*).

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